Chapter Summary

In this chapter we have reviewed differentiation. We've defined the derivative as the instantaneous rate of change of a function, and looked at estimating derivatives using tables and graphs. We've reviewed the formulas for derivatives of basic functions, as well as the product, quotient, and chain rules. We've looked at derivatives of implicitly defined functions and inverse functions, and reviewed two important theorems: Rolle's Theorem and the Mean Value Theorem.

For BC Calculus students, we've reviewed derivatives of parametrically defined functions and the use of L'Hopital's Rule for evaluating limits of indeterminate forms.

Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

In each of Questions 1–20 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

$\frac{d}{dx}(f(x)*g(x)) = f'(x)g(x) + f(x)g'(x)$

1. $y = x^5 \tan x$

(A) $5x^4 \tan x$ **(B)** $x^5 \sec^2 x$ **(C)** $5x^4 \sec^2 x$

(D) $5x^4 + \sec^2 x$ **(E)** $5x^4 \tan x + x^5 \sec^2 x$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ **(B)** $\frac{6x-5}{(3x+1)^2}$ **(C)** $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ **(E)** $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ **(B)** $-\frac{1}{\sqrt{3-2x}}$ **(C)** $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ **(E)** $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ **(B)** $-30(5x+1)^{-4}$ **(C)** $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ **(E)** $\frac{30}{(5x+1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ **(B)** $3x^{-1/3} - 2x^{-1/2}$ **(C)** $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ **(E)** $2x^{-1/3} - 2x^{-1/2}$

6. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

(A) $x + \frac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\frac{4x-1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ **(E)** $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

7. $y = \sqrt{x^2 + 2x - 1}$

(A) $\frac{x+1}{y}$ **(B)** $4y(x+1)$ **(C)** $\frac{1}{2\sqrt{x^2+2x-1}}$

(D) $-\frac{x+1}{(x^2+2x-1)^{3/2}}$ **(E)** none of these

8. $y = \frac{x^2}{\cos x}$

(A) $\frac{2x}{\sin x}$ **(B)** $-\frac{2x}{\sin x}$ **(C)** $\frac{2x\cos x - x^2 \sin x}{\cos^2 x}$

(D) $\frac{2x\cos x + x^2 \sin x}{\cos^2 x}$ **(E)** $\frac{2x\cos x + x^2 \sin x}{\sin^2 x}$

9. $y = \ln \frac{e^x}{e^x - 1}$

(A) $x - \frac{e^x}{e^x - 1}$ **(B)** $\frac{1}{e^x - 1}$ **(C)** $-\frac{1}{e^x - 1}$

(D) 0 **(E)** $\frac{e^x - 2}{e^x - 1}$

10. $y = \tan^{-1} \frac{x}{2}$

(A) $\frac{4}{4+x^2}$ **(B)** $\frac{1}{2\sqrt{4-x^2}}$ **(C)** $\frac{2}{\sqrt{4-x^2}}$

(D) $\frac{1}{2+x^2}$ **(E)** $\frac{2}{x^2+4}$

11. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$

(D) $\dfrac{1}{\sec x + \tan x}$ **(E)** $-\dfrac{1}{\sec x + \tan x}$

12. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$

(D) $\dfrac{4}{(e^x + e^{-x})^2}$ **(E)** $\dfrac{1}{e^{2x} + e^{-2x}}$

13. $y = \ln\left(\sqrt{x^2+1}\right)$

(A) $\dfrac{1}{\sqrt{x^2+1}}$ **(B)** $\dfrac{2x}{\sqrt{x^2+1}}$ **(C)** $\dfrac{1}{2(x^2+1)}$

(D) $\dfrac{x}{x^2+1}$ **(E)** $\dfrac{2x}{x^2+1}$

14. $y = \sin\left(\dfrac{1}{x}\right)$

(A) $\cos\left(\dfrac{1}{x}\right)$ **(B)** $\cos\left(-\dfrac{1}{x^2}\right)$ **(C)** $-\dfrac{1}{x^2}\cos\left(\dfrac{1}{x}\right)$

(D) $-\dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$ **(E)** $\cos(\ln x)$

15. $y = \dfrac{1}{2\sin 2x}$

(A) $-\csc 2x \cot 2x$ **(B)** $\dfrac{1}{4\cos 2x}$ **(C)** $-4 \csc 2x \cot 2x$

(D) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$ **(E)** $-\csc^2 2x$

16. $y = e^{-x}\cos 2x$

(A) $-e^{-x}(\cos 2x + 2\sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x}\sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x}\sin 2x$

17. $y = \sec^2(x)$

(A) $2 \sec x$ **(B)** $2 \sec x \tan x$ **(C)** $2 \sec^2 x \tan x$

(D) $\sec^2 x \tan^2 x$ **(E)** $\tan x$

18. $y = x \ln^3 x$

(A) $\dfrac{3 \ln^2 x}{x}$ **(B)** $3 \ln^2 x$ **(C)** $3x \ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ **(E)** none of these

19. $y = \dfrac{1+x^2}{1-x^2}$

(A) $-\dfrac{4x}{(1-x^2)^2}$ **(B)** $\dfrac{4x}{(1-x^2)^2}$ **(C)** $\dfrac{-4x^3}{(1-x^2)^2}$

(D) $\dfrac{2x}{1-x^2}$ **(E)** $\dfrac{4}{1-x^2}$

20. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\dfrac{1}{2\sqrt{1-x^2}}$ **(B)** $\dfrac{2}{\sqrt{1-x^2}}$ **(C)** $\dfrac{1+x}{\sqrt{1-x^2}}$

(D) $\dfrac{x^2}{\sqrt{1-x^2}}$ **(E)** $\dfrac{1}{\sqrt{1+x}}$

In each of Questions 21–24, y is a differentiable function of x. Choose the alternative that is the derivative $\dfrac{dy}{dx}$.

21. $x^3 - y^3 = 1$

(A) x **(B)** $3x^2$ **(C)** $\sqrt[3]{3x^2}$ **(D)** $\dfrac{x^2}{y^2}$ **(E)** $\dfrac{3x^2-1}{y^2}$

22. $x + \cos(x + y) = 0$

(A) $\csc(x + y) - 1$ **(B)** $\csc(x + y)$ **(C)** $\dfrac{x}{\sin(x+y)}$

(D) $\dfrac{1}{\sqrt{1-x^2}}$ **(E)** $\dfrac{1-\sin x}{\sin y}$

23. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ **(B)** $-\cot y$ **(C)** $\dfrac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ **(E)** $\dfrac{2-\cos x}{\sin y}$

24. $3x^2 - 2xy + 5y^2 = 1$

(A) $\frac{3x+y}{x-5y}$ **(B)** $\frac{y-3x}{5y-x}$ **(C)** $3x + 5y$

(D) $\frac{3x+4y}{x}$ **(E)** none of these

BC ONLY

25. If $x = t^2 + 1$ and $y = 2t^3$, then $\frac{dy}{dx} =$

(A) $3t$ **(B)** $6t^2$ **(C)** $\frac{6t^2}{t^2+1}$ **(D)** $\frac{6t^2}{(t^2+1)^2}$ **(E)** $\frac{2t^4+6t^2}{(t^2+1)^2}$

26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ **(B)** $\{0, -1, -2\}$ **(C)** $\{-1, +2\}$
(D) $\{0\}$ **(E)** $\{0, 1, 2\}$

27. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

(A) -32 **(B)** -16 **(C)** -4 **(D)** -2 **(E)** $-\frac{1}{2}$

28. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\frac{1}{3}$ **(B)** -1 **(C)** -3 **(D)** 1 **(E)** none of these

29. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

(A) 0 **(B)** $\frac{1}{5}$ **(C)** -5 **(D)** $-\frac{1}{5}$ **(E)** nonexistent

BC ONLY

30. If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) 1 **(B)** -1 **(C)** 0 **(D)** 3 **(E)** $\frac{1}{2}$

31. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 **(B)** 1.0 **(C)** 5.0 **(D)** 8.0 **(E)** 32.0

32. If $y = e^x(x - 1)$, then $y''(0)$ equals

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** none of these

BC ONLY

33. If $x = e^\theta \cos\theta$ and $y = e^\theta \sin\theta$, then, when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx}$ is

(A) 1 **(B)** 0 **(C)** $e^{\pi/2}$ **(D)** nonexistent **(E)** -1

BC ONLY

34. If $x = \cos t$ and $y = \cos 2t$, then $\dfrac{d^2y}{dx^2}$ $(\sin t \neq 0)$ is

(A) $4\cos t$ **(B)** 4 **(C)** $\dfrac{4y}{x}$ **(D)** -4 **(E)** $-4\cot t$

35. $\lim\limits_{h\to 0} \dfrac{(1+h)^6 - 1}{h}$ is

(A) 0 **(B)** 1 **(C)** 6 **(D)** ∞ **(E)** nonexistent

36. $\lim\limits_{h\to 0} \dfrac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 **(B)** $\dfrac{1}{12}$ **(C)** 1 **(D)** 192 **(E)** ∞

37. $\lim\limits_{h\to 0} \dfrac{\ln(e+h) - 1}{h}$ is

(A) 0 **(B)** $\dfrac{1}{e}$ **(C)** 1 **(D)** e **(E)** nonexistent

38. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{x}$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** ∞ **(E)** none of these

39. If $f(x) = \begin{cases} \dfrac{4x^2 - 4}{x - 1}, & x \neq 1 \\ 4, & x = 1 \end{cases}$, which of these statements are true?

I. $\lim\limits_{x\to 1} f(x)$ exists.

II. f is continuous at $x = 1$.

III. f is differentiable at $x = 1$.

(A) none **(B)** I only **(C)** I and II only
(D) I and III only **(E)** I, II, and III

40. If $g(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$, which of these statements are true?

I. $\lim\limits_{x\to 3} g(x)$ exists.

II. g is continuous at $x = 3$.

III. g is differentiable at $x = 3$.

(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

41. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined **(B)** $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist **(D)** $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist